Exercise 1:
Let us consider depth-first search of a directed graph. Let $Q$ be the stack which is used for the organization of the search. Show that during the depth-first search the stack $Q$ always contains a simple path from the start node to the top node of $Q$.

Exercise 2:
(a) Develop an algorithm which decides in linear time if a given graph $G = (V, E)$ is bipartite or not.

(b) Develop an algorithm which decides if a given graph $G = (V, E)$ is bipartite or not after the deletion of one edge. What is the run time of your algorithm? Can you give a linear time algorithm for this problem?

Exercise 3:
Construct a graph which contains a maximum matching of size ten and a maximal matching of size five.

Exercise 4:
Let $G = (V, E)$ be an undirected graph, $M$ a maximal and $M'$ a maximum matching of $G$. Prove $|M| \geq \frac{|M'|}{2}$. 