Approximation Taxonomy of Metric TSP

(Update: August 2013)

The list below presents the best up to now known upper and lower approximation bounds for the instances of metric TSP (we refer also to another source on approximation algorithms for metric TSP [8]). It is intended to codify (at one glance) the (many) recent developments and improvements on the approximability of that problem. It is hoped to be useful in further research on the possible improvements of the underlying approximation bounds.

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1 General Metric TSP *

- $\frac{3}{2}$-approximation [6]
- hard to approximate within a factor less then $\frac{123}{122}$ [9]

2 Graphic TSP

- $\frac{7}{5}$-approximation [14]
- hard to approximate within a factor less then $\frac{535}{534}$ [11]

3 (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a factor less then $\frac{535}{534}$ [11]

4 Dense (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a constant $\frac{535}{534}$ [11] [7]

5 Cubic Graphic TSP

- $\frac{4}{3}$-approximation [5], [13]
- hard to approximate within a factor less then $\frac{1153}{1152}$ [10]

*The underlying annotations to the problems can be found in the enclosed references-update.

6 Subcubic Graphic TSP

- $\frac{4}{3}$-approximation [13]
- hard to approximate within a constant $\frac{685}{684}$ [10]

7 Cubic (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a factor less then $\frac{1141}{1140}$ [10]

8 Subcubic (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a constant $\frac{673}{672}$ [10]

9 Asymmetric TSP

- $O(\frac{log\ n}{log\ log\ n})$-approximation [2]
- hard to approximate within a factor less then $\frac{75}{74}$ [9]

10 Asymmetric (1,2)-TSP

- $\frac{5}{4}$-approximation [4]
- hard to approximate within a factor less than $\frac{207}{206}$ [11]

11 Geometric TSP (arbitrary l_p metric)

- PTAS for every fixed dimension [1], [12]
- APX-hard for *log n* dimensions [15]

. $\square$

References-Update

[1] S. Arora. Polynomial Time Approximation Schemes for Euclidean TSP and other Geometric Problems, Journal of the ACM, 45(5):753-782, 1998. 11

[2] A. Asadpour, M. Goemans, A. Madry, S. Gharan, A. Saberi. An O(log n / log log n)-Approximation Algorithm for the Asymmetric Traveling Salesman Problem, Proc. 21st SODA, 379-389, 2010. 9

[3] P. Berman, M. Karpinski. 8/7-Approximation Algorithm for (1-2)-TSP, Proc. 17th SODA, 641-648, 2006. 3, 4, 7, 8

[4] M. Bläser. A 3/4-Approximation Algorithm for Maximum ATSP with Weights Zero and One, Proc. 8th APPROX-RANDOM, LNCS 3122:61-71, 2004. 10

[5] S. Boyd, R. Sitters, S. van der Ster, L. Stougie. The Traveling Salesman Problem on Cubic and Subcubic Graphs, CoRR arXiv:abs/1107.1052, also appeared in Proc. 15th IPCO, LNCS 6655:65-77, 2011. 5

[6] N. Christofides. Worst-case Analysis of a New Heuristic for the Travelling Salesman Problem, Technical Report CS-93-13. Carnegie Mellon University, 1976. (See also on Wikipedia.) 1

[7] B. Csaba, M. Karpinski, P. Krysta. Approximability of Dense and Sparse Instances of Minimum 2-Connectivity, TSP and Path Problems, PProc. of the 13th Annual ACM-SIAM Symposium on Discrete Algorithmics, 74-83, 2002. 4

[8] A. Golovnev. Approximation Algorithms for Metric TSP, Stackexchange.com, URL: http://cstheory.stackexchange.com/questions/9241/approximation-algorithms-for-metric-tsp (document)

[9] M. Karpinski, M. Lampis, R. Schmied. New Inapproximability Bounds for TSP, CoRR arXiv:abs/1303.6437, 2013. 1, 9

[10] M. Karpinski, R. Schmied. Approximation Hardness of Graphic TSP on Cubic Graphs, CoRR arXiv:abs/1304.6800, 2013. 5, 6, 7, 8

[11] M. Karpinski, R. Schmied. On Approximation Lower Bounds with Bounded Metrics, CoRR arXiv:abs/1201.5821, 2012. 2, 3, 4, 10

[12] J. Mitchell. Guillotine Subdivisions Approximate Polygonal Subdivisions: A Simple Polynomial-Time Approximation Scheme for Geometric TSP, k-MST, and Related Problems, SIAM Journal on Computing, 28(4):1298-1309, 1999. 11

[13] T. Mömke, O. Svensson. Approximating Graphic TSP by Matchings, CoRR arXiv:abs/1104.3090, also appeared in 52nd Annual Symposium on Foundations of Computer Science, 560-569, 2011. 5, 6

[14] A. Sebö, J. Vygen. Shorter Tours by Nicer Ears, CoRR arXiv:abs/1201.1870, 2012; To appear in Combinatorica. 2

[15] L. Trevisan. When Hamming Meets Euclid: The Approximability of Geometric TSP and Steiner Tree, SIAM Journal on Computing, 30(2):475-485, 2000. 11