On Approximability of Bounded Degree Instances of Selected Optimization Problems

Richard Schmied

Dept. of Computer Science and
Hausdorff Center for Mathematics,
University of Bonn

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Introduction

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The first problem belongs to the fundamental and most important problems in combinatorial optimization.

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The first problem belongs to the fundamental and most important problems in combinatorial optimization. (formal definition follows later for both problems)

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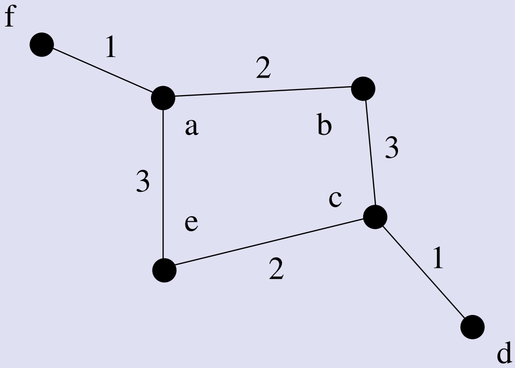
An instance of the Metric TSP (shortest-path metric)



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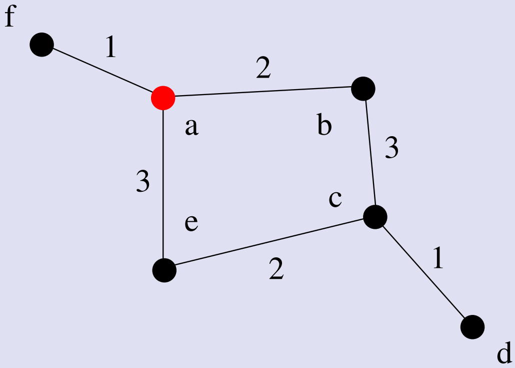

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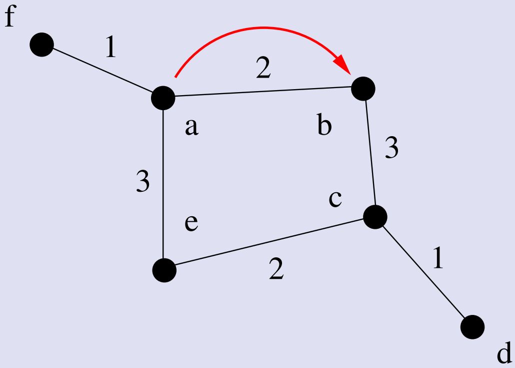

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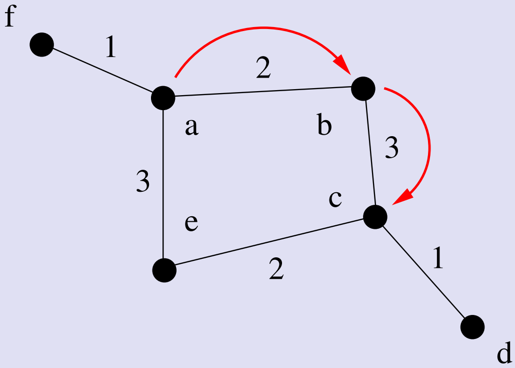

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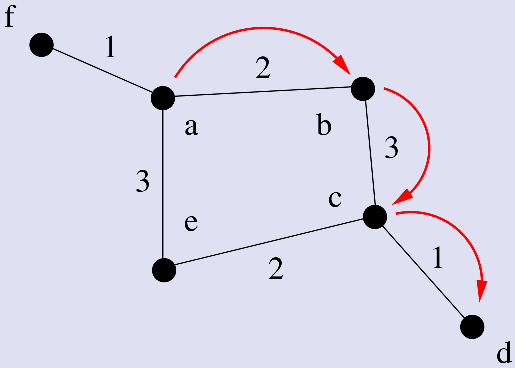

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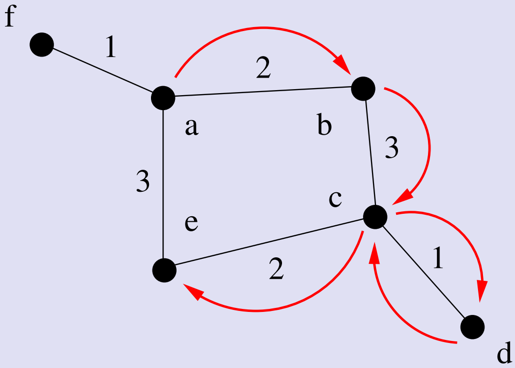

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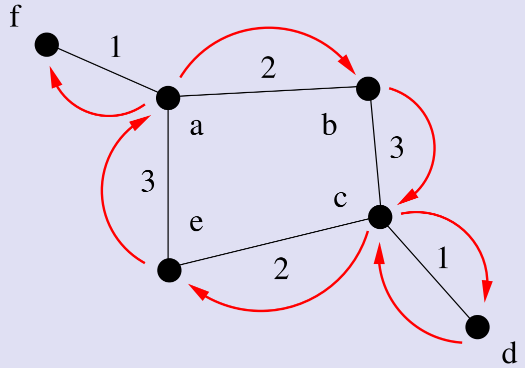

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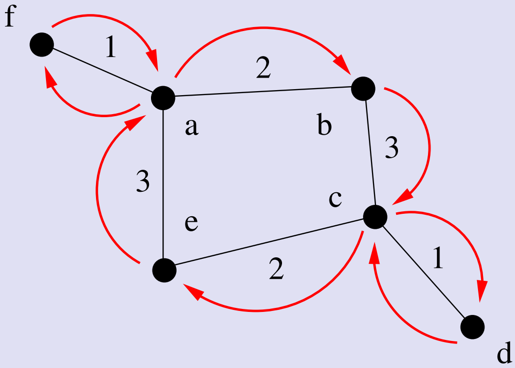

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Definition (Metric TSP)

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Input: A metric space (V, d) (weighted graph, shortest path metric)

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Objective: Find an ordering of the points $v_1, v_2, \ldots, v_n$ such that $d(v_1, v_2) + d(v_2, v_3) + \ldots + d(v_n, v_1)$ is minimized

Metric TSP

The Metric TSP

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- APX-hard [Papadimitriou & Yannakakis'93]
- First explicit inapproximability bound: 5381/5380 [Engebretsen'00]

Metric TSP

Explicit inapproximability constants for the Metric TSP

- 3813/3812 [Böckenhauer et al.'00]

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Theorem (Karpinski, Lampis & S.'13)

It is NP-hard to approximate the Metric TSP within any factor less than 123/122.

The Reduction (Metric TSP)

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Starting point: Inapproximability result for MAX-E3LIN2

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Definition (MAX-E3LIN2)

Input: A system $\mathscr{L}$ of linear equations mod 2, in which equations are of the form $x_i \oplus x_j \oplus x_k = b$ with $b \in \{0, 1\}$

Output: An assignment to the variables in $\mathscr{L}$ that maximizes the number of satisfied equations

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Approximation lower bound: MAX-E3LIN2 is NP-hard to approximate to within any factor less than 2. [Håstad'01]

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High-level view of the reduction:

- Construct a reduction from MAX-E3LIN2 to Metric TSP ($\mathscr{L} \rightarrow$ TSP instance)

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- Construct a reduction from MAX-E3LIN2 to Metric TSP ($\mathscr{L} \rightarrow$ TSP instance)
- Reduction is easier if the number of occurrences of each variable in $\mathscr{L}$ is bounded by a constant (to control the consistency of variable gadgets)

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- Sparse instance methods **(amplifier graphs)** [Berman&Karpinski'99]

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  $\rightarrow$ Prove our result for Metric TSP

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Definition (Strong expander)

A graph $G = (V, E)$ is a **strong expander** if for all $S \subseteq V$ with $|S| \leq |V|/2$, we have that $|\{e \in E \mid |e \cap S| = 1\}| \geq |S|$.

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Construction for reducing the number of occurrences

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Note: $x \oplus y = 0$ if and only if $x = y$ (**equality equation**)

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Optimal assignments are consistent

1. Suppose that in the new instance the optimal assignment sets some of the x_i's to 0 and others to $\mathbf{1}$ $\rightarrow$ partition of the strong expander
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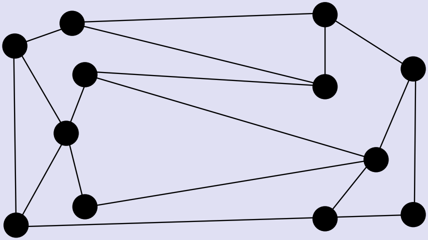

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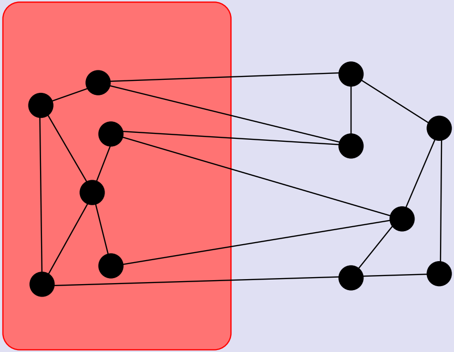

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3. This gives some inapproximability factor

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Sparse Instance Methods

Unfortunately:

For $\Delta < 6$, strong expander are yet not known to exist!

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- The other vertices are thrown in to make consistency easier to achieve (checker vertices)

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- The other vertices are thrown in to make consistency easier to achieve (checker vertices)
- This allows us to get smaller Δ

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Special class of amplifier graphs with $\Delta = 3$:

Wheel amplifier graphs [Berman & Karpinski'99]

Construction:

- Start with a cycle on $7n$ vertices

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- Other vertices are checker vertices
- There is a perfect matching on the set of checker vertices

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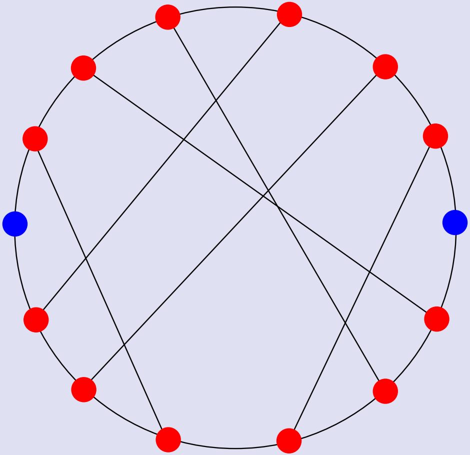
Sparse Instance Methods

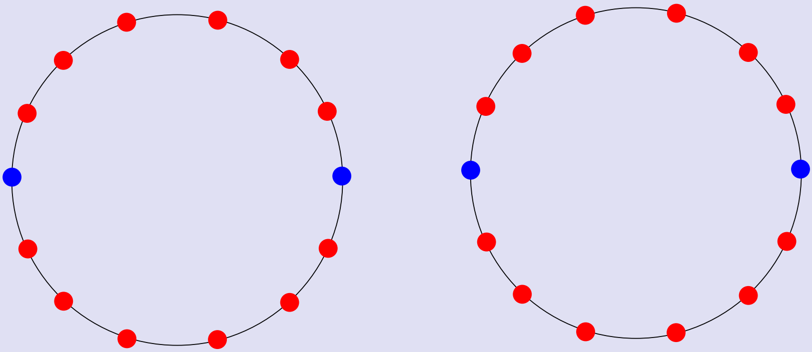
Wheel amplifier graph



Sparse Instance Methods

Amplifier graphs tailored for the Metric TSP:

Bi-wheel amplifier graph [Karpinski, Lampis & S.'13]



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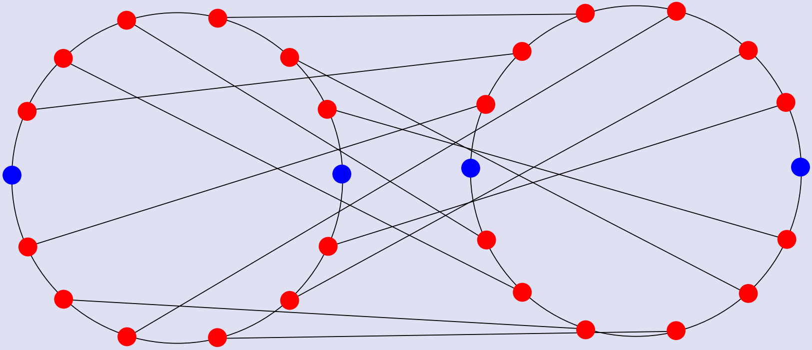

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Note: $x \oplus y = 1$ if and only if $x \neq y$ (**inequality equation**)

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The Reduction (Metric TSP) cont'd

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Construction ($\mathscr{L}_{3occ} \rightarrow$ TSP instance):

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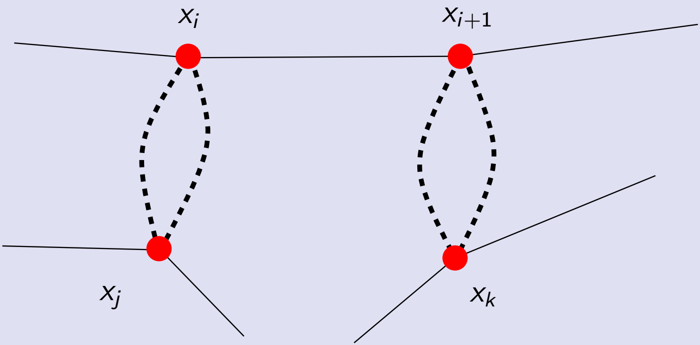
- For each inequality equation, add an **inequality gadget** (will be shown on the next slide)
- Add gadgets for equations with 3 variables (containing the contact vertices)

The Reduction (Metric TSP)

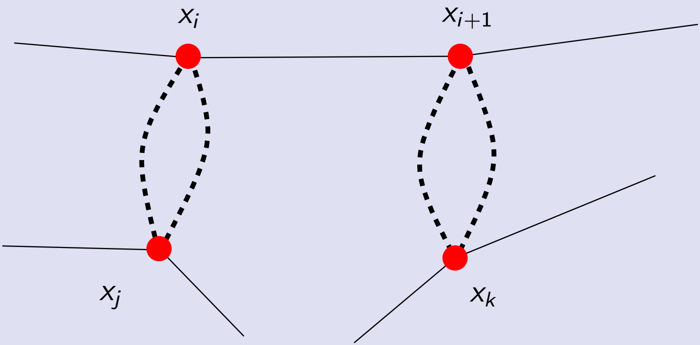
Forced edge: any tour is forced to use this edge at least once

Inequality gadgets for $x_i \oplus x_j = 1$ and $x_{i+1} \oplus x_k = 1$



The Reduction (Metric TSP)

Simulating $x_i \oplus x_{i+1} = 0$, $x_i \oplus x_j = 1$ and $x_{i+1} \oplus x_k = 1$



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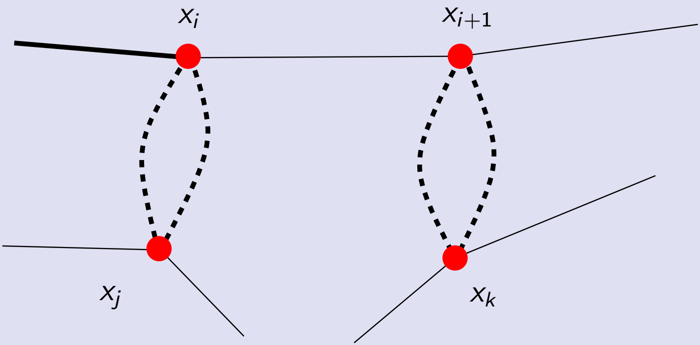

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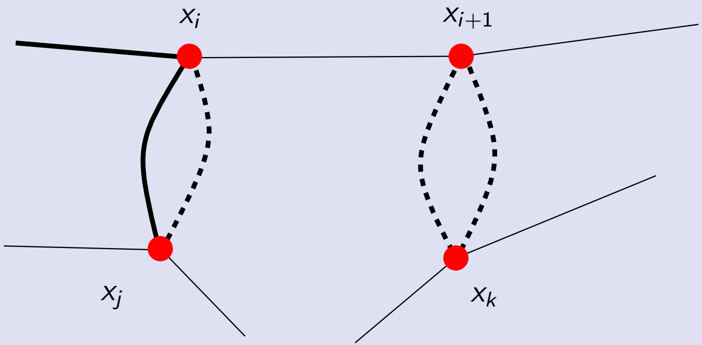

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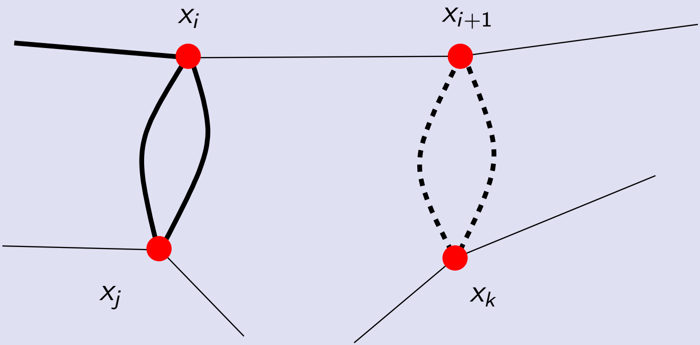

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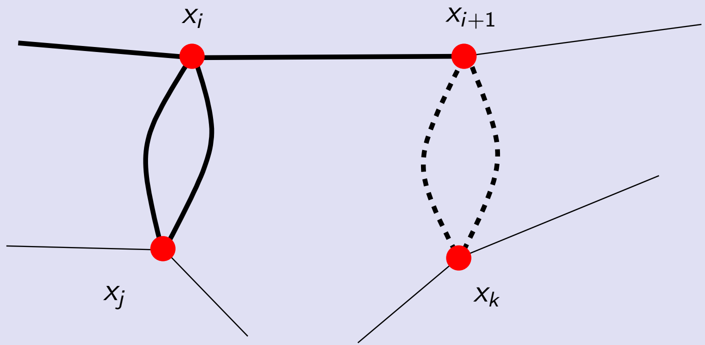

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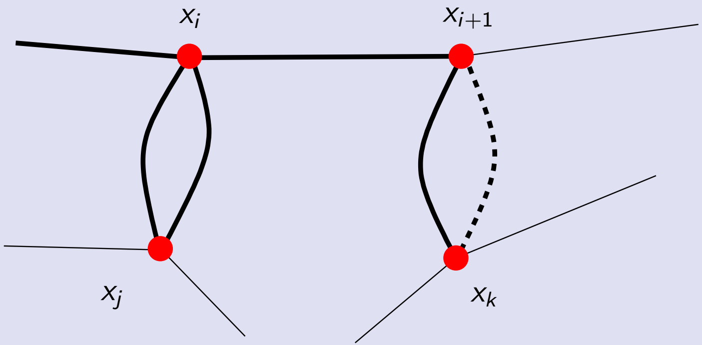

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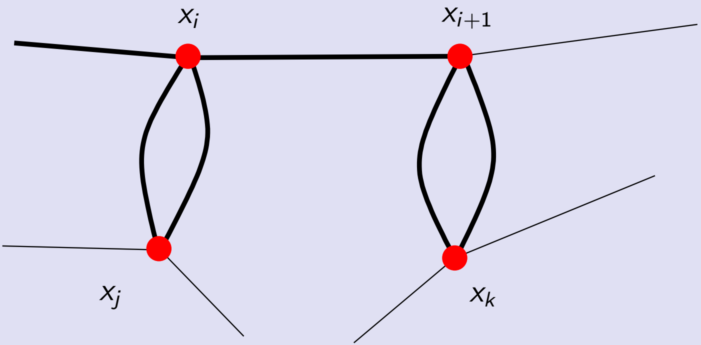

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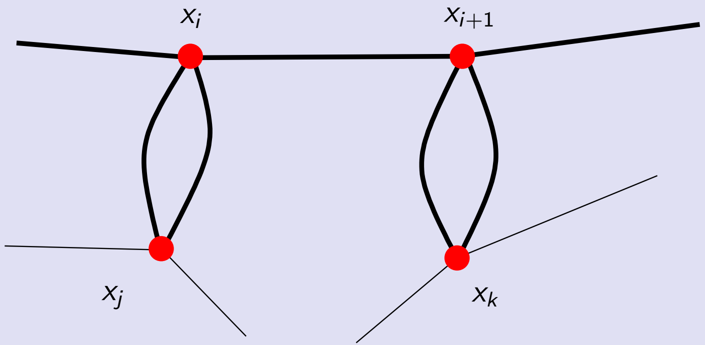

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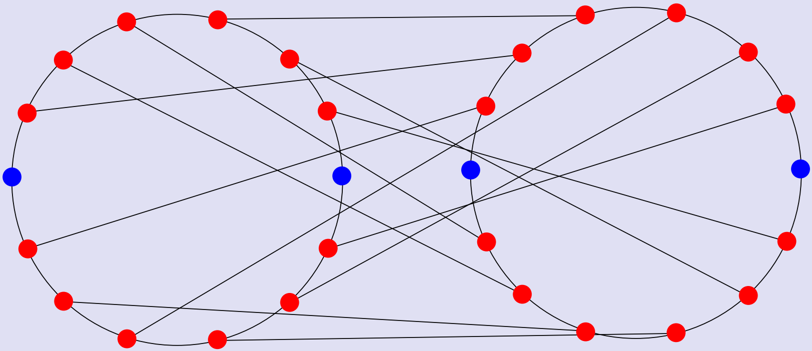
The Reduction (Metric TSP)

Sub-tour visiting all contact vertices of one cycle



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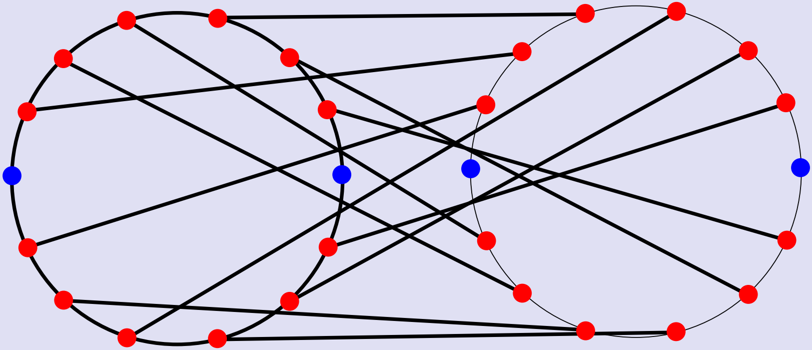

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Final remarks:

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Final remarks:

- For equations with 3 variables, we construct a more efficient gadget (not shown)
- Some work needs to be done to ensure connectivity
- Similar ideas can be used for Asymmetric TSP

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Asymmetric TSP

Asymmetric TSP

Definition (Asymmetric TSP)

Input:

Objective:

Asymmetric TSP

Definition (Asymmetric TSP)

Input: An asymmetric metric space (V, d) (arc weighted digraph)

Objective:

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Asymmetric TSP

Definition (Asymmetric TSP)

Input: An asymmetric metric space (V, d) (arc weighted digraph)

Objective: Find a tour in (V, d) with minimum length

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Asymmetric TSP

Theorem (Papadimitriou & Vempala (STOC'00))

It is NP-hard to approximate the Asymmetric TSP to within any factor less than 117/116.

By using our bi-wheel amplifier methods, we obtain:

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First improvement after more than a decade!

TSP with Distances 1 and 2

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Definition (TSP with distances 1 and 2 ((1,2)-TSP))

Input:

Objective:

TSP with Distances 1 and 2

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Input: A graph $G = (V, E)$

Objective:

TSP with Distances 1 and 2

Definition (TSP with distances 1 and 2 ((1,2)-TSP))

Input: A graph $G = (V, E)$

Objective: Find a tour with minimum length

$d(v_i, v_j) = 1$ if $\{v_i, v_j\} \in E$ and 2 otherwise

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TSP with Distances 1 and 2

Theorem (Engebretsen & Karpinski (ICALP'01))

It is NP-hard to approximate the (1,2)-TSP to within any factor less than 741/740.

By using wheel amplifier graphs combined with "parity gadgets", we obtain:

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TSP with Distances 1 and 2

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By using wheel amplifier graphs combined with "parity gadgets", we obtain:

Theorem (Karpinski & S.'12)

It is NP-hard to approximate the (1,2)-TSP to within any factor less than 535/534.

Graphic TSP

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Input:

Objective:

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Input: A graph $G = (V, E)$

Objective:

Graphic TSP

Definition (Graphic TSP)

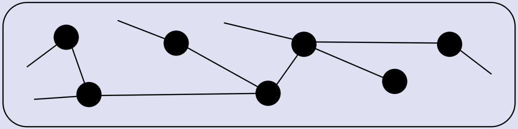
Input: A graph $G = (V, E)$

Objective: Find a tour with minimum length

$d(v_i, v_j)$ is defined by the shortest-path distance of v_i and v_j in G

Graphic TSP

Reducing (1,2)-TSP to Graphic TSP



Graphic TSP

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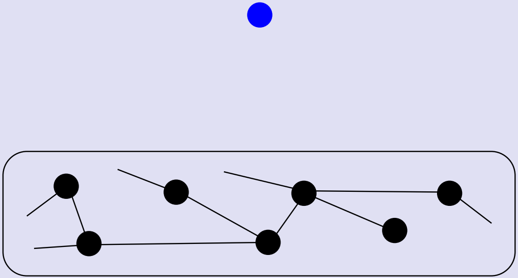

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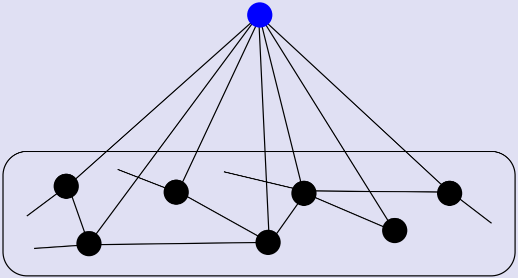

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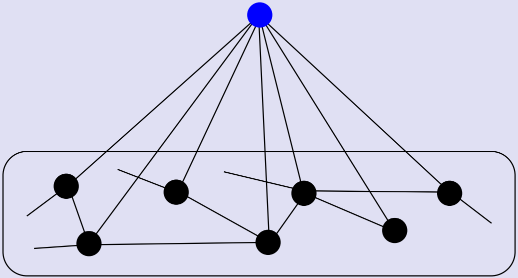

Implying the best up to now inapproximability factor for Graphic TSP

Subcubic & Cubic Instances

Comparison of inapproximability results

Problem	Previously best known	Our Result
(1,2)-TSP on cubic graphs	1291/1290 [CKK02]	
(1,2)-TSP on subcubic graphs	787/786 [CKK02]	
Graphic TSP on cubic graphs	–	
Graphic TSP on subcubic graphs	–	

Subcubic & Cubic Instances

Comparison of inapproximability results

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Graphic TSP on subcubic graphs	–	

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Subcubic & Cubic Instances

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Graphic TSP on cubic graphs	–	
Graphic TSP on subcubic graphs	–	

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Subcubic & Cubic Instances

Comparison of inapproximability results

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Graphic TSP on cubic graphs	–	1153/1152
Graphic TSP on subcubic graphs	–	

First inapproximability results at all!

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Graphic TSP on cubic graphs	–	1153/1152
Graphic TSP on subcubic graphs	–	685/684

First inapproximability results at all!

Further Results

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Further related results on approximability of the problems of:

- Shortest Superstring

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Further related results on approximability of the problems of:

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- Hypergraph Vertex Cover

Conclusions:

- Reduction method for several TSP problems leading to improved inapproximability thresholds

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- Better amplifier constructions?

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- But, the inapproximability constants are still very low!

Further research:

- Improving the inapproximability bounds for the TSP and the Steiner Tree problem?
- Better amplifier constructions?
- New global PCP-system constructions for TSP?

Further research:

On upper bounds side:

- Improving general upper approximation bound for metric TSP below 3/2 (1.50)?

∎

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Further research:

On upper bounds side:

- Improving general upper approximation bound for metric TSP below 3/2 (1.50)?
- Improving upper approximation bound for cubic Graphic TSP below 4/3 (1.33)?

∎

Thank You!

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