ii) a good j enters $L(a_i)$; i.e., there is an agent $a_{i'} \in S(a_i)$ such that the relative utility of the good j becomes to be maximal for $a_{i'}$.

Hence, we have to compute the minimal price increase M_1 such that an edge leaves the layered network $\mathcal{G}(\bar{p}, a_i)$ and also the minimal price increase M_2 such that a new good enters the layered network $\mathcal{G}(\bar{p}, a_i)$. Then

$$M := \min \{ M_1, M_2, r(e_i) \}$$

is the total price increase chosen for the current round.

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<u>The computation of M_1:</u>

The agent a_i has to pay the total price increase. Hence, for each good $j \in L(a_i)$ his price increase has to be transported from j to a_i through $\mathcal{G}(\bar{p}, a_i)$.

Let L_r be the last layer which contains goods. The price increase of the goods in L_r has to be payed by the agents in layer S_r.

(1) Each good j in layer L_r sends to the agents $a_{i'}$ in layer S_r with $(a_{i'}, j) \in E(a_{i'})$ a request

$$r(j, a_{i'}).$$

such that

$$r(j) := \sum_{a_{i'} : (a_{i'}, j) \in E(a_{i'})} r(j, a_{i'}) = \frac{p_j}{\tilde{p}} \cdot X$$

where X is the total price increase of all goods in $L(a_i)$.

(2) Each agent $a_{i'}$ in layer S_r has to pay all requests obtained from goods in L_r. Since $a_{i'}$ is saturated, he has to reduce the amount of money used for paying goods in layer L_{r-1} such that he can pay all requests.

(3) For each good j in layer L_{r-1} let

$$\text{red}(j)$$

be the total reduction of the agents in layer S_r with respect to the good j.

The good j sends to the agents $a_{i'}$ in layer S_{r-1} with $(a_{i'}, j) \in E(a_{i'})$ a request

$$r(j, a_{i'})$$

such that

$$r(j) := \sum_{a_i:(a_i,j)\in E_{r-1r-1}} r(j, a_i) = red(j) + \frac{p_j}{\tilde{p}} \cdot X$$

(4) Each agent a_i in Layer S_{r-1} has to pay all requests obtained from goods in L_{r-1}. Since a_i is saturated, he has to reduce the amount of money used for paying goods in layer L_{r-2} such that he can pay all requests.

(5) For each good j in layer L_{r-2} let

$$red(j)$$

be the total reduction of the agents in layer S_{r-1} with respect to good j.

$\vdots$ a.s.o.

We have to answer the following questions:

i) How to distribute the total request $r(j)$ of a good j to the agents connected to j?

ii) For agent a_i let $r(a_i)$ denote sum of all requests which a_i has to pay. How to distribute $r(a_i)$ to the goods buyed by a_i.

For a formal definition, we need some notations:

For a good $j \in L_e$, we define

- $E_{in}(j) := \{ (a_{i'}, j) \mid a_{i'} \in S_{e+1} \text{ and } (a_{i'}, j) \in E(a_{i'}) \}$

and

- $E_{out}(j) := \{ (a_{i'}, j) \mid a_{i'} \in S_e \text{ and } (a_{i'}, j) \in E(a_{i'}) \}$

For an agent $a_{i'} \in S_e$, we define

- $E_{in}(a_{i'}) := \{ (a_{i'}, j) \mid j \in L_e \text{ and } (a_{i'}, j) \in E(a_{i'}) \}$

and

- $E_{out}(a_{i'}) := \{ (a_{i'}, j) \mid j \in L_{e-1} \text{ and } (a_{i'}, j) \in E(a_{i'}) \}$

Furthermore

$m(a_{i'}, j)$ denotes the amount of money used by agent $a_{i'}$ for buying good j.

and

$$m_{out}(a_{i'}) := \sum_{(a_{i'}, j) \in E_{out}(a_{i'})} m(a_{i'}, j)$$

Obviously, $r(a_{i'})$ cannot be larger than $m_{out}(a_{i'})$ since $m_{out}(a_{i'})$ is the largest possible reduction of money for agent $a_{i'}$.

Idea:

Distribute $r(a_{i'})$ proportionally to edges $(a_{i'}, j) \in E_{out}(a_{i'})$. This means that the edge $(a_{i'}, j)$ obtains

$$\frac{m(a_{i'}, j)}{m_{out}(a_{i'})} \cdot r(a_{i'}).$$

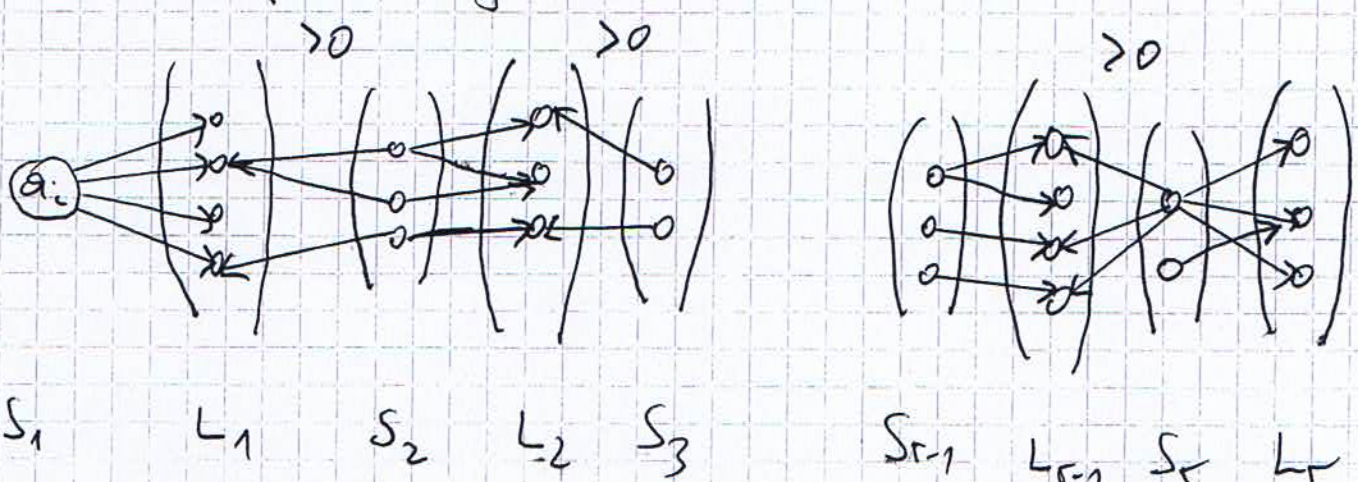
Then all edges in $E_{out}(a_{i'})$ leave the layered network or all edges remain to be an edge in the layered network. All edges in $E_{out}(a_{i'})$ leave the layered network iff $r(a_{i'}) = m_{out}(a_{i'})$.

Let $M(a_{i'})$ denote the total price increase such that $r(a_{i'}) = m_{out}(a_{i'})$.

Goal:

The computation of $M(a_{i'})$ $\forall a_{i'} \in S(a_i)$.

Structure of the layered network:



We start in the last layer L_r. For each good j in L_r, we get the expression for his price increase by $\frac{p_j}{\tilde{p}} X$

The good j has to distribute his price increase $\frac{p_j}{\tilde{p}} \cdot x$ to the agents $a_{i'} \in S_r$ such that $(a_{i'}, j) \in E_{out}(j)$. For doing this we need a fixed rule. (for all layers)

Examples :

i) Apply a rule for choosing an unique agent $a_{i'}$ with $(a_{i'}, j) \in E_{out}(j)$. This agent obtains the whole price increase $\frac{p_j}{\tilde{p}} x$. All other agents obtains nothing.

ii) Distribute the price increase $\frac{p_j}{\tilde{p}} \cdot x$ equally to the agents.

$$\Rightarrow \quad a_{i'} \text{ obtains } \frac{1}{|E_{out}(j)|} \cdot \frac{p_j}{\tilde{p}} \cdot X .$$

♦

Starting in the last layer, we compute the values $M(a_{i'})$ layer by layer.

Let S_ℓ, $\ell > 1$ be the current layer. Assume that

1) for each agent $a_{i'} \in S_\ell$, an expression for $F(a_{i'})$ is computed. Note that the expression depends on X.

2) our goal is the computation of $M(a_{i'})$ $\forall\, a_{i'} \in S_\ell$.

For $\ell = r$ note that

$$r(a_{i'}) = \sum_{(a_{i'}, j) \in E_{in}(a_{i'})} r(j, a_{i'})$$

Consider the equation

$$r(a_{i'}) = m_{out}(a_{i'})$$

We isolate x and obtain the unique solution $M(a_{i'})$.

Next we have to compute for each $a_{i''} \in S_{\ell-1}$ an expression for $r(a_{i''})$. This is done as follows:

(1) Compute for each $j \in L_{\ell-1}$ an expression for $r(j)$.

$\rightsquigarrow$

$$r(j) = \underbrace{\frac{p_j}{\beta}}_{\text{price increase of } j} \cdot x + \sum_{(a_{i'}, j) \in E_{in}(j)} \frac{m(a_{i'}, j)}{m_{out}(a_{i'})} \cdot \underbrace{r(a_{i'})}_{\text{expression for } r(a_{i'})}$$

(2) Apply the fixed rule for obtaining an expression

$$r(j, a_{i''}) \quad \forall\, a_{i''} \text{ with } (a_{i''}, j) \in E_{out}(j).$$

(3) We obtain an expression for $r(a_{i''})$, $a_{i''} \in S_{\ell-1}$

$$r(a_{i''}) := \sum_{(a_{i''}, j) \in E_{in}(a_{i''})} r(j, a_{i''}).$$

After the computation of $M(a_{i'})$ for all $a_{i'} \in S(a_i)$, we obtain M_1 by

$$M_1 := \min \{ M(a_{i'}) \mid a_{i'} \in S(a_i) \}.$$

<u>The computation of M_2</u>

Consider $a_{i'} \in S(a_i)$. Let

$\alpha_{i'}$ be the size of the maximal relative utility for $a_{i'}$

and

$\beta_{i'}$ be the size of the largest relative utility $< \alpha_{i'}$ for $a_{i'}$ with respect to goods not in $L(a_{i'})$.

Because of the increase of the prices of the goods in $L(a_i)$ the maximal relative utilities for the agents in $S(a_i)$ decreases.

At the moment when $\alpha_{i'} = \beta_{i'}$ we have to add some goods to $L(a_i)$ and to extend the layered network.

Let

$\bar{M}(a_{i'})$ denote the unique total price increase such that $\alpha_{i'} = \beta_{i'}$.

As observed above, we obtain

$$\bar{M}(a_{ij}) = \left(\frac{\alpha_{ij}}{\beta_{ij}} - 1\right)\tilde{p}$$

After the computation of $M'(a_{ij})$ for all $a_{ij} \in S(a_i)$, we obtain

$$M_2 := \min\left\{ \bar{M}(a_{ij}) \;\middle|\; a_{ij} \in S(a_i) \right\}.$$

In dependence on

$$M := \min\left\{ M_1, M_2, r(a_i) \right\}$$

we compute the new price vector

$$\bar{q} := (q_1, q_2, \dots, q_m).$$

and compute the corresponding layered network.

If $r(e_i) = 0$ then a_i is saturated. Otherwise, we repeat the whole.

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