$$\bar{M}(a_{ij}) = \left(\frac{\alpha_{ij}}{\beta_{ij}} - 1\right)\tilde{p}$$

After the computation of $M'(a_{ij})$ for all $a_{ij} \in S(a_{ij})$, we obtain

$$M_2 := \min\{\bar{M}(a_{ij}) \mid a_{ij} \in S(a_{ij})\}.$$

In dependence on

$$M := \min\{M_1, M_2, r(a_{ij})\}$$

we compute the new price vector

$$\bar{q} := (q_1, q_2, \ldots, q_m).$$

and compute the corresponding layered network.

If $r(e_i) = 0$ then a_i is saturated. Otherwise, we repeat the whole.

10.07

Next, we have to prove the correctness of the algorithm and to analyze the used time. The following theorem shows the correctness of the algorithm if the algorithm terminates.

Theorem 8.3

If the algorithms for the computation of a market equilibrium for a Fisher market terminates, then the computed price vector is a market equilibrium.

Proof:

If the algorithm terminates then all agents in B are saturated. This means that each agent uses his whole endowment. Since each agent buys only goods which have maximal relative utility for him, all agents buy an optimal bundle of goods. It remains to prove that all goods $j \in A$ are sold out.

By the construction of the algorithm, there hold:

i) If a good becomes sold out then the good remains to be sold out during the whole algorithm.

ii) If a good is not sold out then his price is the initial price of the good.

Assume that after the termination of the algorithm there is a good $j \in B$ which is not sold out.

$\Rightarrow$

His price p_j is the initial price of the good j.

Construction of the initial price vector $\Rightarrow$

There is an agent a_i such that the good j has maximal relative utility for a_i with respect to the initial price vector.

Since the algorithm increases only prices of some goods it follows:

At the moment when the algorithm starts with the saturation of the agent a_i the good j is of maximal relative utility for agent a_i

$\Longrightarrow$

Agent a_i buys the portion of good j which is not buyed by another agent.

$\Longrightarrow$

The good j becomes sold out.

Observation $i)$ $\Rightarrow$

The good j remains to be sold out, a contradiction

■

Next we shall investigate the time analysis. Let us consider SATURATE(a_i), $i \geq 1$.

- Computation of the relative utilities of the goods in B for agent a_i:

  - can be done in time $O(m)$

- Computation of $G(a_i)$ and buying everything in $G(a_i)$ which is not already buyed by another agent

  - can be done in time $O(m)$

- Construction of the layered network $\mathcal{G}(a_i, \bar{p})$
  - can be done in time $O(m \cdot n)$
- If there are some goods in $\mathcal{G}(a_i, \bar{p})$ which are not sold out then
  - apply the algorithm of Karzanov to reach at least one of the following situations
    - a blocking flow is computed such that all goods which are not sold out leave the layered network
    - all goods in $\mathcal{G}(a_i, \bar{p})$ become sold out
  - can be done in $O(m \cdot n)$ time

    This has to be worked out!
- Computation of M_1
  - can be done in time $|\mathcal{G}(a_i, \bar{p})| = O(m \cdot n)$
- Computation of M_2
  - can easily be done in time $O(m \cdot n)$ (perhaps better!)

    This has to be worked out!
- Computation of M
  - time $O(1)$

- Computation of the new price vector
  $\bar{p}^* = (p_1^*, p_2^*, \dots, p_m^*)$

  – time $O(m)$.

Altogether, one round of can be performed in $O(n \cdot m)$ time.

If D is an upper bound for the number of rounds performed by SATURATE (a_i) then SATURATE(a_i) uses $O(D \cdot n \cdot m)$ time

$\rightsquigarrow$

We have to investigate for a proof of an upper bound D for the number of rounds which is as small as possible.

For getting such an upper bound, we have to analyze the situations which occur.

12.07