

To prove that

$$M' = M \oplus M' \oplus M$$

consider

$$M \oplus M' = \underbrace{(M \cup M')}_{\bar{M}} \setminus (M \cap M')$$

and

$$\bar{M} \oplus M = (\bar{M} \cup M) \setminus (\bar{M} \cap M).$$

Since

$$\bar{M} \cup M = (M \cup M') \setminus (M \cap M') \cup M = M' \cup M$$

and

$$\bar{M} \cap M = (M \cup M') \setminus (M \cap M') \cap M = M \setminus (M \cap M')$$

we obtain

$$\bar{M} \oplus M = (M' \cup M) \setminus (M \setminus (M \cap M')) = M'.$$

To prove that

$$w(M \oplus M' \oplus M) \leq w(M)$$

consider the connected components of  $G'$   
which are not an isolated node.

Note that

$$w(M) = w(M \setminus (M \cap M')) + w(M \cap M')$$

and

$$w(M') = w(M' \setminus (M \cap M')) + w(M \cap M')$$

Since for each connected component  $P$   
of  $G'$  which is not an isolated node

$$\Delta(P) \leq 0$$

there holds

$$\omega(M' \setminus (M \cap M')) \leq \omega(M \setminus (M \cap M'))$$

and hence,

$$\begin{aligned}\omega(M') &= \omega(M' \setminus (M \cap M')) + \omega(M \cap M') \\ &\leq \omega(M \setminus (M \cap M')) + \omega(M \cap M') \\ &= \omega(M).\end{aligned}$$